Notiuni teoretice

Distanta dintre doua puncte in plan

Distanta dintre punctele $M_1(x_1, y_1), M_2(x_2, y_2)$ este

$$M_1M_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Panta unei drepte oblice **Dreptele verticale nu au panta!**

(1) $m = tg\theta, \quad \theta \in [0, \pi) \setminus \left\{\frac{\pi}{2}\right\}$, unde θ este unghiul facut de dreapta cu semiaxa pozitiva Ox.

(2) $m = \frac{y_2-y_1}{x_2-x_1}, \quad x_1 \neq x_2$ unde $M_1(x_1, y_1), M_2(x_2, y_2)$ sunt 2 puncte ale ei.

(3) $m = \frac{b}{a}, \quad a \neq 0,$ unde $\vec{v} = a\vec{i} + b\vec{j}$ este vectorul director al dreptei.

Diverse ecuatii ale dreptei in plan

(1) Ecuatia dreptei determinata de punctul $M_0(x_0, y_0)$ si panta m:

$$y - y_0 = m(x - x_0)$$

(2) Ecuatia dreptei determinata de doua puncte distincte $M_1(x_1, y_1), M_2(x_2, y_2)$:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}, \quad x_1 \neq x_2, y_1 \neq y_2.$$

Daca $x_1 = x_2 \Rightarrow x = x_1$.

Daca $y_1 = y_2 \Rightarrow y = y_1$.

(3) Ecuatia explicita

$$y = mx + n,$$

n este ordonata la origine

(4) Ecuatia dreptei determinata de punctul $M_0(x_0, y_0)$ vectorul director $\vec{v} = a\vec{i} + b\vec{j}, a \neq 0$:

$$y - y_0 = \frac{b}{a}(x - x_0)$$

(5) Ecuatia dreptei prin taieturi $M_1(a, 0), M_2(0, b)$ (punctele de intersectie cu Ox respectiv Oy):

$$\frac{x}{a} + \frac{y}{b} = 1, \quad a \neq 0, b \neq 0.$$

(6) Ecuatia generala:

$$ax + by + c = 0, \quad a^2 + b^2 \neq 0 \quad (\text{adica } a \neq 0 \text{ sau } b \neq 0)$$

(7) Ecuatia dreptei determinata de punctele distincte $M_1(x_1, y_1), M_2(x_2, y_2)$, sub forma de determinant:

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

(8) Cazuri particulare:

(i) Ecuatia axei Ox : $\quad y = 0$

(ii) Ecuatia axei Oy : $\quad x = 0$

(iii) Ecuatia unei drepte orizontale: $\quad y = ct$

(iv) Ecuatia unei drepte verticale: $\quad x = ct$

Conditia de coliniaritate e trei puncte $M_1(x_1, y_1), M_2(x_2, y_2), M_3(x_3, y_3)$

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Aria triunghiului $M_1M_2M_3,\ M_1(x_1, y_1), M_2(x_2, y_2), M_3(x_3, y_3)$

$$Aria = \frac{1}{2} \left| \Delta \right|,$$

unde

$$\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Distanta de la punctul $M_0(x_0, y_0)$ **la dreapta** $d_1 : ax + by + c = 0$

$$d(M_0, d_1) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

Pozitia a doua drepte in plan

Cazul I. Dreptele sunt date sub forma explicita

$$d_1 : y = m_1 x + n_1$$

$$d_2 : y = m_2 x + n_2$$

(1)

$$d_1 \| d_2 \iff \begin{cases} m_1 = m_2 \\ n_1 \neq n_2 \end{cases}$$

(2)

$$d_1 \perp d_2 \iff m_1 m_2 = -1$$

(3)

$$d_1 = d_2 \iff \begin{cases} m_1 = m_2 \\ n_1 = n_2 \end{cases}$$

Cazul II. Dreptele sunt date sub forma generala

$$d_1 : a_1 x + b_1 y + c_1 = 0$$

$$d_2 : a_2 x + b_2 y + c_2 = 0$$

(1)

$$d_1 \| d_2 \iff \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

(2)

$$d_1 \perp d_2 \iff a_1 a_2 + b_1 b_2 = 0$$

(3)

$$d_1 = d_2 \iff \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Ecuatiile bisectoarelor unui unghi Fie laturile unghiului:

$$d_1 : a_1 x + b_1 y + c_1 = 0$$

$$d_2 : a_2 x + b_2 y + c_2 = 0$$

Bisectoarea este locul geometric al punctelor din plan egal departate de cele doua laturi.

Fie $M(x, y) \in$bisectoarei $\iff d(M, d_1) = d(M, d_2) \Rightarrow$

$$\frac{|a_1 x + b_1 y + c_1|}{\sqrt{a_1^2 + b_1^2}} = \frac{|a_2 x + b_2 y + c_2|}{\sqrt{a_2^2 + b_2^2}}$$

Deci ecuatiile bisectoarelor sunt:

$$\frac{a_1 x + b_1 y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2 x + b_2 y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Unghiul dintre 2 drepte

$$d_1 : y = m_1 x + n_1 (m_1 = tg\theta_1),$$

$$d_2 : y = m_2 x + n_2, (m_2 = tg\theta_2),$$

$\theta \in [0, \frac{\pi}{2})$ unghiul dintre cele doua drepte este

$$tg\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

Problema 543

Problema 543

$\boxed{C}$ $\boxed{1}$ Întrebare: Fie punctele $A(\lambda, 1), B(2,3), C(3,-1)$. Sa se determine λ astfel incat punctul A sa se afle pe dreapta determinata de punctele B si C.

$\boxed{A}$ 2 $\boxed{B}$ 3 $\boxed{C}$ $\frac{5}{2}$ $\boxed{D}$ $\frac{1}{2}$ $\boxed{E}$ $\frac{2}{3}$

$\Leftrightarrow$ A, B, C coliniare $\Leftrightarrow \Delta = 0$ unde $\Delta = \begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix}$

$$\begin{vmatrix} \lambda & 1 & 1 \\ 2 & 3 & 1 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

$$3\lambda - 2 + 3 - 9 + \lambda - 2 = 0$$

$$4\lambda - 10 = 0$$

$$\lambda = \frac{10}{4}$$

$$\lambda = \frac{5}{2} \Rightarrow \boxed{C}$$

Problema 544

Problema 544

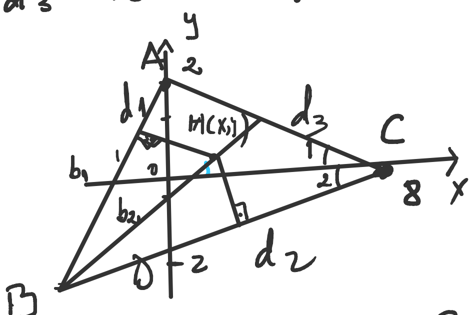
$\boxed{\boxed{A}}$ $\boxed{2}$ Întrebare: Dreptele $4x - y + 2 = 0, x - 4y - 8 = 0, x + 4y - 8 = 0$ determina un triunghi Centrul cercului inscris in triunghi este

$\boxed{A}$ $(\frac{6}{5}, 0)$ $\boxed{B}$ $(\frac{6}{5}, 1)$ $\boxed{C}$ $(\frac{5}{6}, 0)$ $\boxed{D}$ $(\frac{5}{6}, 1)$ $\boxed{E}$ $(\frac{6}{5}, \frac{5}{6})$

Fie $d_1 = AB: 4x-y+2=0$ $\begin{cases} x=0 \Rightarrow y=2 \Rightarrow (0,2) \\ y=0 \Rightarrow 4x=-2 \Rightarrow x=-\frac{1}{2} \Rightarrow (-\frac{1}{2},0) \end{cases}$

$d_2 = BC: x-4y-8=0$ $\begin{cases} x=0 \Rightarrow -4y-8=0 \Rightarrow y=-2 \Rightarrow (0,-2) \\ y=0 \Rightarrow x=8 \Rightarrow (8,0) \end{cases}$

$d_3 = AC: x+4y-8=0$ $\begin{cases} x=0 \Rightarrow 4y=8 \Rightarrow y=2 \Rightarrow (0,2) \\ y=0 \Rightarrow x=8 \Rightarrow (8,0) \end{cases}$



Centrul cerc. inscris in Δ: la $\cap$ bisectoarelor

Obs. că: bisect. $\measuredangle C$ este $OC: y=0$ (not b_1 această bisectoare)

($\Delta AOC \equiv \Delta DOC$ – dr, C.C.)

$\Downarrow$

$\measuredangle C_1 = \measuredangle C_2$

Not b_2 bisectoarea $\measuredangle B$, $\{B\} = d_1 \cap d_2$, $d_1: 4x-y+2=0$, $d_2: x-4y-8=0$

Fie $M(x,y) \in b_2 \Rightarrow d(M,d_1) = d(M,d_2)$

$$\frac{|4x-y+2|}{\sqrt{17}} = \frac{|x-4y-8|}{\sqrt{17}}$$

$$|4x-y+2| = |x-4y-8|$$

$$4x-y+2 = \pm(x-4y-8)$$

1° $4x-y+2 = x-4y-8$

$3x+3y+10=0$

$3y=-3x-10$

$y = \underbrace{-}_{m=-1}x - \frac{10}{3}$

2° $4x-y+2 = -(x-4y-8)$

$5x-5y-6=0$

$5y=5x-6$

$y = \underbrace{\,}_{m=1>0}x - \frac{6}{5}$

$\measuredangle$ format de b_2 cu Ox este ascuțit $\Rightarrow m_{b_2} > 0$

$\Rightarrow b_2: 5x-5y-6=0$

$b_1 \cap b_2$: $\begin{cases} b_1: y=0 \\ b_2: 5x-5y-6=0 \end{cases} \overset{y=0}{\Rightarrow} 5x=6 \Rightarrow x=\frac{6}{5} \Rightarrow \left(\frac{6}{5}, 0\right) \Rightarrow \boxed{A}$

Problema 545

Problema 545

$\boxed{D}$ $\boxed{3}$ Întrebare: Triunghiul ABC are latura $[AB]$ pe dreapta $4x + y - 8 = 0$, latura $[AC]$ pe dreapta $4x + 5y - 24 = 0$, iar varfurile B si C pe axa Ox. Ecuatia medianei corespunzatoare varfului A este:

$\boxed{A}$ $2x + 3y = 0$ $\boxed{B}$ $3x + 2y = 0$ $\boxed{C}$ $5x + y = 9$ $\boxed{D}$ $4x + 3y - 16 = 0$ $\boxed{E}$ $x + 4y - 17 = 0$

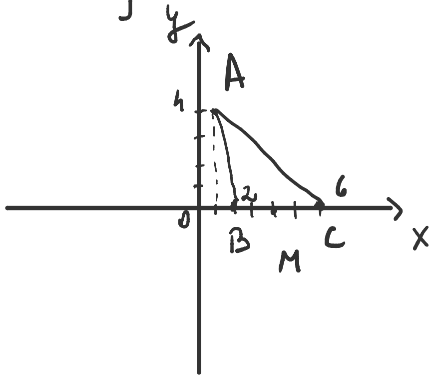
Det. coord pct. A. Adica rez. sistemul format din ec. dr. AB, AC

$$\begin{aligned} AB:&\ \left\{\begin{array}{l} 4x+y-8=0 \ \ |\cdot(-1) \\ 4x+5y-24=0 \end{array}\right. \Rightarrow \left\{\begin{array}{l} -4x-y+8=0 \\ 4x+5y-24=0 \end{array}\right. \\ AC:&\ \end{aligned}$$

$$4y-16=0 \Rightarrow y=4 \Rightarrow x=1 \Rightarrow A(1,4)$$

$AB: 4x+y-8=0,\ B \in Ox \Rightarrow y_B = 0 \Rightarrow 4x-8=0 \Rightarrow x=2 \Rightarrow B(2,0)$

$AC: 4x+5y-24=0,\ C \in Ox \Rightarrow y_C = 0 \Rightarrow 4x-24=0 \Rightarrow x=6 \Rightarrow C(6,0)$



Fie M mijl $[BC] \Rightarrow \left\{\begin{array}{l} x_M = \frac{x_B+x_C}{2} = \frac{2+6}{2} = 4 \\ y_M = \frac{y_B+y_C}{2} = 0 \end{array}\right.$

$\Rightarrow M(4,0)$

$A(1,4)$

AM : mediana $\frac{x-x_A}{x_M-x_A} = \frac{y-y_A}{y_M-y_A} \Rightarrow \frac{x-1}{4-1} = \frac{y-4}{-4}$

$\Rightarrow \frac{x-1}{3} = \frac{y-4}{-4} \Rightarrow -4x+4 = 3y-12$

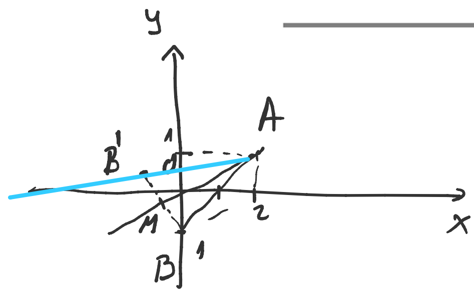
$4x+3y-16=0 \Rightarrow \boxed{D}$

Problema 546

Problema 546

$\boxed{\text{E}}$ $\boxed{4}$ Întrebare: Se dau punctele $A(2,1), B(0,-1)$. Ecuatia simetricei dreptei AB fata de dreapta OA este:

$\boxed{\text{A}}$ $x+2y-1=0$ $\boxed{\text{B}}$ $3x-7y+1=0$ $\boxed{\text{C}}$ $2x+y+5=0$ $\boxed{\text{D}}$ $x+y+1=0$ $\boxed{\text{E}}$ $x-7y+5=0$



Determinăm simetricele pct A, B fața de Ox

$A \in OA \Rightarrow$ simetricul pct A fața de OA e A

Determinăm simetricul pt B fața de OA, not B'

Fie $BB' \perp OA$, $M \in OA$, M mijl. $[BB']$

$OA: \frac{x}{2} = \frac{y}{1} \Rightarrow 2y = x = y = \underbrace{\frac{1}{2}}_{m_{OA}} x \Rightarrow m_{OA} = \frac{1}{2}$

$BB' \perp OA \Rightarrow m_{BB'} \cdot \underbrace{m_{OA}}_{\frac{1}{2}} = -1 \Rightarrow m_{BB'} \cdot \frac{1}{2} = -1 \Rightarrow m_{BB'} = -2$

$BB': y - y_B = m(x - x_B)$, $\left.\begin{matrix} m = m_{BB'} = -2 \\ B(0,-1) \end{matrix}\right\} \Rightarrow$

$\Rightarrow y+1 = -2x \Rightarrow y = -2x-1$

$\{M\} = OA \cap BB' \Rightarrow \begin{matrix} OA: \\ BB': \end{matrix} \begin{cases} y = \frac{1}{2}x \\ y = -2x-1 \end{cases} \Rightarrow \frac{1}{2}x = -2x-1$

$x = -4x-2$

$5x = -2 \Rightarrow x = -\frac{2}{5} \Rightarrow y = -\frac{1}{5}$

$\Rightarrow M\left(-\frac{2}{5}, -\frac{1}{5}\right)$

M mijl $BB' \Rightarrow \begin{cases} x_M = \frac{x_B + x_{B'}}{2} \\ y_M = \frac{y_M + y_{M'}}{2} \end{cases} \Rightarrow \begin{cases} -\frac{2}{5} = \frac{0 + x_{B'}}{2} \\ -\frac{1}{5} = \frac{-1 + y_{B'}}{2} \end{cases} \Rightarrow \begin{cases} x_{B'} = \frac{-4}{5} \\ y_{B'} = \frac{3}{5} \end{cases} \Rightarrow B'\left(-\frac{4}{5}, \frac{3}{5}\right)$

$A(2,1)$, $B'\left(-\frac{4}{5}, \frac{3}{5}\right)$

$AB': \frac{x-2}{-\frac{4}{5}-2} = \frac{y-1}{\frac{3}{5}-1} \Rightarrow \frac{x-2}{\frac{-14}{5}} = \frac{y-1}{\frac{-2}{5}} \Rightarrow \frac{x-2}{7} = \frac{y-1}{1} \Rightarrow x - 7y + 5 = 0$

$\Rightarrow \boxed{E}$

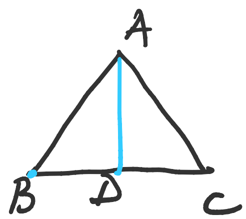
Pt. a găsi ec. simetricei dr. (d) fața de un punct M sau fața de o dr. (d_1) este suficient să găsim coordonatele simetricelor a două puncte ale dr. (d) fața de M respectiv (d_1)

Problema 547

Problema 547

$\boxed{\boxed{A}}$ $\boxed{5}$ Întrebare: Fie triunghiul ABC unde $B(-4,-5)$. Ecuatia inaltimii duse din A este $5x+3y-4$. Ecuatia dreptei BC este:

$\boxed{A}$ $5y-3x+13=0$ $\boxed{B}$ $3x-5y+37=0$ $\boxed{C}$ $y=-5$ $\boxed{D}$ $x+y-2=0$ $\boxed{E}$ $y-2x=3$



Fie AD înălț. din A.

$AD \perp BC \Rightarrow m_{AD} \cdot m_{BC} = -1 \Rightarrow -\frac{5}{3} \cdot m_{BC} = -1$

$m_{BC} = \frac{3}{5}$

$AD: 5x+3y-4=0$

$3y = -5x+4$

$y = \underbrace{-\frac{5}{3}}_{m_{AD}} x + \frac{4}{3}$

$BC: B(-4,-5), m_{BC} = \frac{3}{5}$

$y - y_B = m_{BC}(x - x_B)$

$y+5 = \frac{3}{5}(x+4)$

$5y+25 = 3x+12$

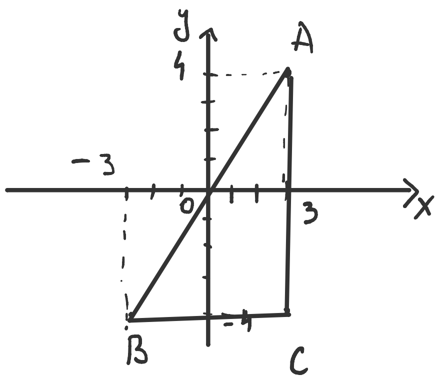
$-3x+5y+13=0 \Rightarrow \boxed{A}$

Problema 548

Problema 548

$\boxed{C}$ $\boxed{\mathbf{6}}$ Întrebare: In sistemul cartezian xOy se considera punctele $A(3,4), B(-3,-4), C(3,-4)$ Coordonatele centrului circumscris triunghiului ABC sunt:

$\boxed{A}$ $(1,1)$ $\boxed{B}$ $(-1,0)$ $\boxed{C}$ $(0,0)$ $\boxed{D}$ $(0,1)$ $\boxed{E}$ $(0,-1)$



Centrul cerc. circumscris: la $\cap$ mediatoarelor

$\underline{M_I}$ ΔABC dr. $\Rightarrow$ centrul cerc. circ. este în mijl. ipot $\Rightarrow$

$\Rightarrow$ O este centrul cerc. circumscris

$\underline{M_{II}}$ mediatoarea laturii AC : Ox $\}$ $\Rightarrow$ O este centrul

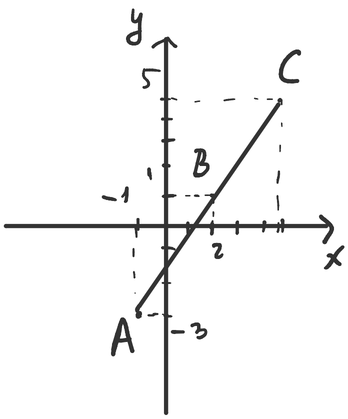
—"— —"— — BC : Oy $\}$ cerc. circ.

Problema 549

Problema 549

$\boxed{\text{A}}$ $\boxed{\mathbf{7}}$ Întrebare: Fie C simetricul punctului $A(-1,-3)$ fata de punctul $B(2,1)$ Care sunt coordonatele punctului C?

$\boxed{\text{A}}$ $(5,5)$ $\boxed{\text{B}}$ $(4,5)$ $\boxed{\text{C}}$ $(6,5)$ $\boxed{\text{D}}$ $(5,6)$ $\boxed{\text{E}}$ $(4,6)$



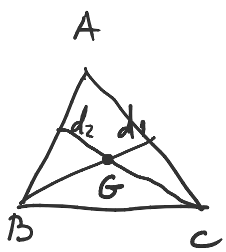
$$B \text{ mij } [AC] \Rightarrow \begin{cases} x_B = \dfrac{x_A + x_C}{2} \\ y_B = \dfrac{y_A + y_C}{2} \end{cases} \Rightarrow \begin{cases} 2 = \dfrac{-1 + x_C}{2} \\ 1 = \dfrac{-3 + y_C}{2} \end{cases} \Rightarrow \begin{cases} x_C = 5 \\ y_C = 5 \end{cases}$$

$$\Rightarrow C(5,5) \Rightarrow \boxed{A}$$

Problema 551

$\boxed{\boxed{A}}$ $\boxed{\mathbf{9}}$ Întrebare: Fie $A(0,-1), d_1 : x - y + 1 = 0, d_2 : 2x - y = 0$. Coordonatele punctelor $B \in d_1$ si $C \in d_2$ pentru care d_1 si d_2 sunt mediane in triunghiul ABC sunt:

$\boxed{A}$ $(0,1),(3,6)$ $\boxed{B}$ $(0,1),(0,1)$ $\boxed{C}$ $(-1,0),(1,1)$ $\boxed{D}$ $(0,0),(-1,1)$ $\boxed{E}$ $(-1,-1),(1,1)$



Obs.: $A \notin d_1$

$A \notin d_2$

$\{G\} = d_1 \cap d_2$

centrul de greutate

$$\begin{cases} d_1: x-y+1=0 \\ d_2: 2x-y=0 \Rightarrow y=2x \end{cases} \Rightarrow x-2x+1=0 \Rightarrow -x+1=0 \Rightarrow x=1 \Rightarrow G(1,2)$$

! $B \in d_1: x-y+1=0 \Rightarrow y=x+1 \Rightarrow B(x_B, x_B+1)$!

! $C \in d_2: 2x-y=0 \Rightarrow y=2x \Rightarrow C(x_C, 2x_C)$

$$\begin{cases} x_G = \dfrac{x_A+x_B+x_C}{3} \\ y_G = \dfrac{y_A+y_B+y_C}{3} \end{cases} \Rightarrow \begin{cases} 1 = \dfrac{0+x_B+x_C}{3} \\ 2 = \dfrac{\cancel{-1}+x_B\cancel{+1}+2x_C}{3} \end{cases} \Rightarrow \begin{cases} x_B+x_C=3 \\ x_B+2x_C=6 \end{cases} \Rightarrow \begin{matrix} x_B=0 \Rightarrow y_B=1 \\ x_C=3 \Rightarrow y_C=6 \end{matrix}$$

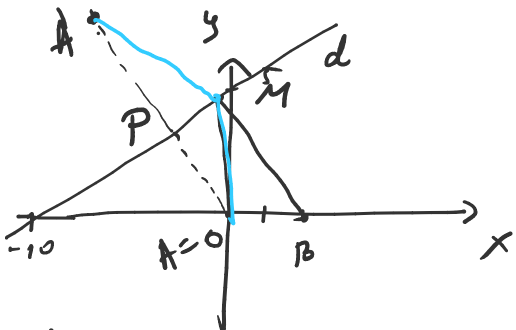
$\Rightarrow \boxed{A}$

Problema 554

Problema 554

$\boxed{B}$ $\boxed{13}$ Întrebare: Se considera in plan punctele $A(0,0), B(2,0)$ si dreapta de ecuatie d : $x-2y+10=0$. Valoarea minima sumei $S(M) = MA + MB$, cand punctul M parcurge dreapra d este:

$\boxed{A}$ 2 $\boxed{B}$ 10 $\boxed{C}$ $\sqrt{101}$ $\boxed{D}$ $\sqrt{98}$ $\boxed{E}$ $7\sqrt{2}$



$A(0,0) = O(0,0)$

$d: x-2y+10=0 \quad \begin{cases} x=0 \Rightarrow y=5 \\ y=0 \Rightarrow x=-10 \end{cases}$

! Fie A' simetricul lui A fata de d, $M(x,y)\in d$) $\Rightarrow MA=MA'$

Dar: $MA+MB = MA'+MB \to \min \Rightarrow A', M, B$ coliniare !

$d: x-2y+10=0 \Rightarrow y = \frac{1}{2}x+5 \Rightarrow m_d = \frac{1}{2}$

$AA' \perp d \Rightarrow m_{AA'}\cdot m_d = -1 \Rightarrow m_{AA'} = -2$

$\begin{matrix} AA': \\ d: \end{matrix} \begin{cases} y=-2x \\ x-2y+10=0 \end{cases} \Rightarrow \begin{matrix} x=-2 \\ y=4 \end{matrix} \Rightarrow P(-2,4)$

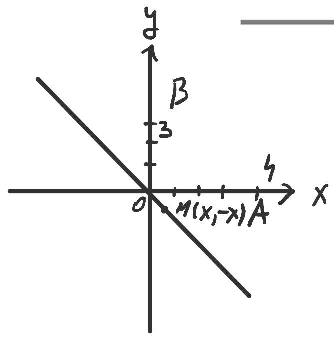
P e mijl $AA' \Rightarrow \left.\begin{matrix} A'(-4,8) \\ B(2,0) \end{matrix}\right\} \Rightarrow A'B = \sqrt{36+64} = \sqrt{100} = 10$

Problema 543, UT 2018

Problema 543, 2018

$\boxed{A}$ $\boxed{\mathbf{13}}$ Întrebare: Fie dreapta $(d): x + y = 0$ si punctele $A(4,0), B(0,3)$. Valoarea minima a sumei $MA^2 + MB^2$ cand punctul M parcurge dreapra d este:

$\boxed{A}$ $\frac{99}{4}$ $\boxed{B}$ 25 $\boxed{C}$ $\frac{101}{4}$ $\boxed{D}$ $\sqrt{26}$ $\boxed{E}$ $\frac{105}{4}$



$(d): x+y=0 \Rightarrow y=-x$ (a doua bisectoare)

$M \in (d) \Rightarrow M(x,-x)$

$MA^2 = (x-4)^2 + x^2 = 2x^2 - 8x + 16$

$MB^2 = x^2 + (x+3)^2 = 2x^2 + 6x + 9$

$MA^2 + MB^2 = \underbrace{4x^2 - 2x + 25}_{\text{not } f(x)} \to \min$, $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 4x^2 - 2x + 25$

$f'(x) = 8x - 2$

$f'(x) = 0 \Rightarrow x = \frac{1}{4}$

$f\left(\frac{1}{4}\right) = \frac{99}{4}$

x		$\frac{1}{4}$	
$f'(x)$	$-$	0	$+$
$f(x)$	$\searrow$	$\frac{99}{4}$	$\nearrow$

$\Rightarrow$ valoarea minimă $= \frac{99}{4}$

Problema simulare 2016

Enunţ problemă: Fie punctul $A(3,5)$ si multimea $\mathcal{B} = \{(x,y) \in \mathbb{R}^2 \mid |x| + |y| = 2\}$

$\boxed{A}$ $\boxed{18}$ Întrebare: Numarul punctelor multimii $\mathcal{B}$ situate pe Ox este:

$\boxed{A}$ 2 $\boxed{B}$ 3 $\boxed{C}$ 4 $\boxed{D}$ 0 $\boxed{E}$ 1

$\boxed{A}$ $\boxed{19}$ Întrebare: Multimea $\mathcal{B}$ este:

$\boxed{A}$ un patrat $\boxed{B}$ un triunghi $\boxed{C}$ o dreapta $\boxed{D}$ un cerc $\boxed{E}$ un segment de dreapta

$\boxed{A}$ $\boxed{20}$ Întrebare: Distanta minima de la punctul A la punctele multimii $\mathcal{B}$ este:

$\boxed{A}$ $3\sqrt{2}$ $\boxed{B}$ 7 $\boxed{C}$ $2\sqrt{3}$ $\boxed{D}$ $5\sqrt{2}$ $\boxed{E}$ $2\sqrt{2}$

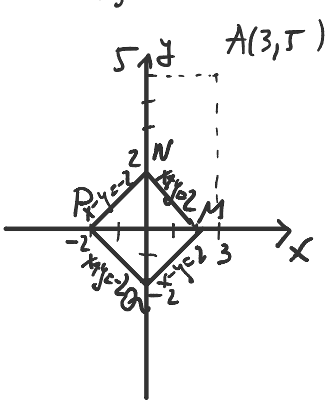
$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$, $|y| = \begin{cases} y, & y \geq 0 \\ -y, & y < 0 \end{cases}$

1) $x \geq 0, y \geq 0 \Rightarrow x + y = 2$ $\begin{cases} x = 0 \Rightarrow y = 2 \Rightarrow (0,2) \\ y = 0 \Rightarrow x = 2 \Rightarrow (2,0) \end{cases}$

2) $x \geq 0, y < 0 \Rightarrow x - y = 2$ $\begin{cases} x = 0 \Rightarrow y = -2 \Rightarrow (0,-2) \\ y = 0 \Rightarrow x = 2 \Rightarrow (2,0) \end{cases}$

3) $x < 0, y \geq 0 \Rightarrow -x + y = 2 \Rightarrow x - y = -2$ $\begin{cases} x = 0 \Rightarrow y = 2 \Rightarrow (0,2) \\ y = 0 \Rightarrow x = -2 \Rightarrow (-2,0) \end{cases}$

4) $x < 0, y < 0 \Rightarrow -x - y = 2 \Rightarrow x + y = -2$ $\begin{cases} x = 0 \Rightarrow y = -2 \Rightarrow (0,-2) \\ y = 0 \Rightarrow x = -2 \Rightarrow (-2,0) \end{cases}$



$\boxed{18}$ 2 pct. pe $Ox \Rightarrow \boxed{A}$

$\boxed{19}$ Mult. $\mathcal{B}$ este un patrat $\Rightarrow \boxed{A}$

$\boxed{20}$ $d(A, \mathcal{B}) = d(A, MN) = \dfrac{|3+5-2|}{\sqrt{2}} = \dfrac{6}{\sqrt{2}} = 3\sqrt{2} \Rightarrow \boxed{A}$